

Taking the temperature of a black hole

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ABSTRACT: We use the global embedding of a black hole spacetime into a higher dimensional flat spacetime to define a local temperature for observers in free fall outside a static black hole. The local free-fall temperature remains finite at the event horizon and in asymptotically flat spacetime it approaches the Hawking temperature at spatial infinity. Freely falling observers outside an AdS black hole do not see any high-temperature thermal radiation even if the Hawking temperature of such black holes can be arbitrarily high.

KEYWORDS: Black Holes, Models of Quantum Gravity.

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1. Introduction

A black hole in asymptotically flat spacetime emits thermal radiation with characteristic temperature

$$T_H = \frac{\kappa}{2\pi}, \tag{1.1}$$

where κ is the surface gravity of the black hole [1]. The Hawking temperature T_H is the temperature of the radiation as measured by asymptotic observers. The local temperature measured at a finite distance from the black hole will in general be different from T_H and depends on the state of motion of the observer carrying out the measurement. For example, the temperature measured by a fiducial observer, i.e. an observer who remains at rest with respect to the black hole at a fixed distance, will differ from the temperature measured by an observer in free fall at the same distance.

For a spherically symmetric static black hole, described by a line element of the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2, \tag{1.2}$$

the local fiducial temperature is given by

$$T_{\text{FID}}(r) = \frac{T_H}{\sqrt{f(r)}}. \tag{1.3}$$

The fiducial temperature T_{FID} diverges at the black hole event horizon and in asymptotically flat spacetime T_{FID} approaches the Hawking temperature asymptotically far away from the black hole. In asymptotically anti de Sitter spacetime, on the other hand, the fiducial temperature goes to zero far away from the black hole.

There are several ways to obtain T_{FID} . It can, for example, be inferred from the transition rate of a particle detector interacting with the radiation field at a fixed position with respect to the black hole, as was shown by Unruh [2]. In the same paper, Unruh also

showed that a uniformly accelerated observer in flat spacetime, with proper acceleration a , will detect thermal radiation at the so-called Unruh temperature,

$$T_U = \frac{a}{2\pi}. \quad (1.4)$$

The Hawking effect and the Unruh effect are related. Both arise from a mismatch between vacuum states employed by different observers and both involve regions of spacetime that are hidden behind horizons.¹ The connection between the Unruh effect and the Hawking effect is particularly apparent for a large Schwarzschild black hole, for which the near-horizon region is almost flat. In this case, a fiducial observer who remains fixed close to the event horizon measures high fiducial temperature (1.3), but this observer is also highly accelerated in (almost) flat spacetime.

There is also a more indirect relation between the two effects that involves the global embedding of the black hole spacetime into a higher dimensional flat spacetime [4, 5]. In this context the Hawking effect for a fiducial observer in the black hole spacetime can be interpreted as the Unruh effect for a uniformly accelerated observer in the higher dimensional flat spacetime. This relationship has been mapped out for a variety of static, spherically symmetric spacetime geometries by Deser and Levin [5] and will be briefly reviewed below. The local fiducial temperature and the Unruh temperature in a flat embedding geometry have also been shown to match for various d -dimensional static black holes in [6].

The main goal of the present paper is, however, to obtain the local temperature as measured by observers in free fall towards a black hole. The notion of a local free-fall temperature is less precise than that of a fiducial temperature. This is because a detector in free fall is not static but moves in an environment that is a function of the radial variable r and will therefore in general measure a spectrum of radiation that is not precisely thermal. On the other hand, for a macroscopic black hole and an observer who is moving sufficiently slowly with respect to the black hole rest frame the change in the environment will be slow compared to the microscopic processes involved inside a particle detector, allowing temperature readings to be made as the observer progresses along the free fall orbit. Even if these temperature readings do not correspond to a precise thermal state they provide an effective temperature that indicates the amount of heating that an observer in free fall would experience along a given orbit. See [7] for a related discussion involving observers in general stationary accelerated frames in flat spacetime.

We obtain the local free-fall temperature by extending the above embedding method to a certain class of freely falling observers. Our proposal involves two key steps. One is to consider the Unruh effect for accelerated observers whose proper acceleration is not uniform but changes with time. As long as the rate of change of the acceleration is slow on the timescale of temperature measurements with whatever particle detector a given observer is carrying we may talk about a local Unruh temperature being measured along the observer's orbit.

The other key step is to extend the embedding method to include a certain class of observers in free fall, which we refer to as 'freely falling at rest', or FFAR, observers. As

¹The two effects are not equivalent, however. In particular, the quantum states that enter into the two calculations are different (for a review see [3]).

emphasized by Deser and Levin [5] the matching between the local temperature measured by a fiducial observer and the Unruh temperature of the corresponding uniformly accelerated observer in the higher-dimensional flat spacetime rests on the world-line of the fiducial observer being tangent to a time-like Killing vector in the black hole spacetime. Or, in more physical terms, that the fiducial observer is at rest in a static geometry. The fact that geodesic orbits are not tangent to the time-like Killing vector suggests that the embedding method cannot be applied to observers in free fall but we note that there are special points where the tangent vector to a geodesic orbit is in fact parallel to the time-like Killing vector. These are the turning points of radial geodesics where freely falling observers are momentarily at rest with respect to the black hole. Such a 'freely-falling observer at rest', or FFAR observer, can be obtained by switching off the acceleration keeping a fiducial observer in place. Immediately after the release the formerly fiducial observer is in free fall but has not yet begun moving towards the black hole.

The FFAR condition at a given spatial location outside the event horizon determines a unique radial orbit that has its turning point at that location. The radial orbit in turn defines a curve in the higher dimensional flat embedding spacetime and this curve can be viewed as the worldline of an accelerated observer in the flat spacetime. In general the acceleration of this observer is not uniform but for a macroscopic black hole we will have a local Unruh temperature defined at the point on the curve that corresponds to the turning point of the radial orbit, and it is this local Unruh temperature that we identify as the local free fall temperature in the black hole geometry. We note that it seems natural to consider FFAR observers when discussing the black hole temperature as measured in free fall because they are precisely those freely falling observers who are at rest, if only momentarily, in the black hole rest frame. As discussed above, even FFAR observers will detect a spectrum of radiation that is only approximately thermal. All other observers in free fall are moving with respect to the source of the Hawking radiation and therefore detect a spectrum of radiation that is further 'transformed' away from thermal.

We consider a number of examples in subsequent sections of the paper to illustrate the procedure and to check if it provides physically reasonable answers for a local free-fall temperature around various four-dimensional black holes including Schwarzschild, AdS-Schwarzschild, and Reissner-Nordström. The case of so-called large AdS-Schwarzschild black holes is particularly interesting since the definition (1.1) gives a Hawking temperature that grows without bound with increasing black hole mass, but in recent work by S. Hemming and one of us [8] it was claimed that observers in free fall would nevertheless measure very low ambient temperature near one of these large AdS black holes. This expectation is confirmed by the direct calculation of a free-fall temperature for AdS-Schwarzschild in section 4.

It would be interesting to compare our results obtained by the embedding method to a more conventional calculation involving an detector coupled to a quantum field in a black hole background geometry [2]. The fact that an observer who falls into a black hole is not stationary, and runs into the black hole singularity within a finite proper time, makes it awkward to extract the detector transition rate per unit time from such a calculation. There is, however, another class of freely falling observers for which a conventional computation

of the Hawking effect can be carried out, namely geodesic observers in stable circular orbits around a spherically symmetric black hole. Such a calculation was performed by Chen *et al.* in [9]. They also applied the embedding method to this class of observers and compared results obtained by the two methods. A circular orbit around a Schwarzschild black hole is mapped into a curve that involves rotation in addition to acceleration in the 5+1 dimensional flat embedding spacetime. A generalized Unruh effect involving motion along a general stationary curve of this type in 3+1 dimensional Minkowski spacetime was studied by Korsbakken and Leinaas in [7]. It is straightforward to extend their results to higher dimensions and Chen *et al.* found a perfect match between the local temperature measured by a geodesic observer in a circular orbit around a Schwarzschild black hole and the generalized Unruh temperature of the corresponding observer in the higher-dimensional embedding space,

$$T_{\text{CIRC}}(r) = \frac{T_H}{\sqrt{1 - 3m/r}}, \tag{1.5}$$

where m is the black hole mass. We will not repeat their calculations here but refer to [7] and [9] for details.

In the present paper we are primarily interested in observers in radial free fall into a black hole. As mentioned above, such observers have the drawback that they are not undergoing stationary motion but since they eventually fall into the black hole they probe the near horizon region where there are no stable circular orbits.

2. Higher-dimensional embedding

Let S be a d dimensional spacetime manifold with metric $g_{\mu\nu}$ that is embedded into a D dimensional Minkowski spacetime M with a metric η_{IJ} of mostly plus signature and with one or more time dimensions. Here the Greek indices run from 0 to $d - 1$ and the capital Roman indices from 0 to $D - 1$, $D > d$. Writing the embedding functions,

$$Z^I = Z^I(x^\mu) \tag{2.1}$$

the metric of S is induced from the higher-dimensional flat metric through

$$g_{\mu\nu} = \eta_{IJ} \frac{\partial Z^I}{\partial x^\mu} \frac{\partial Z^J}{\partial x^\nu}. \tag{2.2}$$

Such an embedding always exists [10] but obtaining a global embedding in the presence of an event horizon requires some care.

The tangent space $T(M)$ of the Minkowski spacetime, with basis $\{\frac{\partial}{\partial z^I}\}$, can be written $T(M) = T(S) \oplus N(S)$, where $T(S)$ is the tangent space of S , with basis $\{\frac{\partial}{\partial x^\mu}\}$, and $N(S)$ is orthogonal to $T(S)$. We choose a basis $\{\hat{\xi}_m\}$ in $N(S)$ such that it is orthonormal with respect to the Minkowskian metric of M , i.e. $\eta(\hat{\xi}_m, \hat{\xi}_n) = \eta_{mn}$. In the examples we consider below $N(S)$ is either Euclidean or has one time-like dimension.

If $x^\mu(\tau)$ is a timelike curve in S then the tangent vector $u = \frac{d}{d\tau}$ can be written [10, 11]

$$\frac{du}{d\tau} = \nabla_u u + \alpha(u, u). \tag{2.3}$$

Here ∇_u denotes the covariant derivative in S along u and α is the second fundamental form of S . We have that $\nabla_u u \in T(S)$ and $\alpha(u, u) \in N(S)$ so that when we square equation (2.3) we obtain the Pythagorean relation,

$$a_D^2 = a_d^2 + \alpha^2 \tag{2.4}$$

where

$$\begin{aligned} a_D^2 &= \eta_{IJ} a_D^I a_D^J, & \frac{du}{d\tau} &= a_D^I \frac{\partial}{\partial z^I}, \\ a_d^2 &= g_{\mu\nu} a_d^\mu a_d^\nu, & \nabla_u u &= a_d^\mu \frac{\partial}{\partial x^\mu}, \\ \alpha^2 &= \eta_{mn} \alpha^m \alpha^n, & \alpha(u, u) &= \alpha^m \hat{\xi}_m. \end{aligned} \tag{2.5}$$

The a_D^I are the components of the acceleration vector of an observer in the D -dimensional embedding space moving along the image of the original curve while a_d^μ are the components of the acceleration in S .

In the case of a fiducial observer in a static spherically symmetric spacetime S the corresponding D -dimensional observer is uniformly accelerated. If the constant D -acceleration is spacelike it defines an Unruh temperature which agrees with the local fiducial temperature in S [5]. If, on the other hand, the D -acceleration is timelike the Unruh temperature is formally purely imaginary and the fiducial observer in S does not detect any thermal radiation. An example of the latter behavior is provided by fiducial observers in an otherwise empty AdS spacetime, i.e. observers who are accelerated so as to remain in a fixed position relative to a reference observer at the origin in AdS space [5].

For FFAR observers we find the image under the embedding of the turning point of the corresponding radial orbit, evaluate the acceleration D -vector there, and take the resulting local Unruh temperature in M to define a local temperature for observers in free fall in S . We consider several different static spherically symmetric geometries of the form (1.2) and verify that the free-fall temperature so obtained meets a number of criteria that can be expected on physical grounds. In particular, the free-fall temperature is finite at the event horizon of a black hole while the fiducial temperature is divergent. This follows immediately from the embedding relation (2.4) since $a_d = 0$ for an observer in free fall while for a fiducial observer $a_d \rightarrow \infty$ at the event horizon.

3. Schwarzschild black holes

Consider a four-dimensional Schwarzschild black hole. The line element is given by (1.2) with

$$f(r) = 1 - \frac{2m}{r}, \tag{3.1}$$

where m is the black hole mass. A global embedding of the Schwarzschild geometry into six-dimensional Minkowski spacetime with metric $\eta_{IJ} = \text{diag}(-1, 1, \dots, 1)$ was found by

Fronsdal [12],

$$\begin{aligned}
 Z^0 &= 4m \sqrt{1 - 2m/r} \sinh(t/4m), \\
 Z^1 &= 4m \sqrt{1 - 2m/r} \cosh(t/4m), \\
 Z^2 &= \int dr \sqrt{2m/r + 4m^2/r^2 + 8m^3/r^3}, \\
 Z^3 &= r \sin \vartheta \cos \varphi, \\
 Z^4 &= r \sin \vartheta \sin \varphi, \\
 Z^5 &= r \cos \vartheta.
 \end{aligned}
 \tag{3.2}$$

Now consider a freely falling observer, dropped from rest at $\tau = 0$ at $r = r_0$. The equations for the orbit are

$$\frac{dt}{d\tau} = \frac{\sqrt{1 - x_0}}{1 - x}, \quad \frac{dx}{d\tau} = \frac{x^2}{2m} \sqrt{x - x_0},
 \tag{3.3}$$

where we have introduced a dimensionless radial variable $x \equiv 2m/r$ which runs from $x = 0$ at $r \rightarrow \infty$ to $x = 1$ at the event horizon. The 6-acceleration is spacelike for all timelike orbits and at the turning point $x = x_0$, where the observer is dropped from rest, its magnitude is given by

$$a_6 = \frac{1}{4m} \sqrt{1 + x + x^2 + x^3}.
 \tag{3.4}$$

Taking the local temperature measured by a freely falling observer at rest to be the local Unruh temperature of the corresponding observer in the six-dimensional flat spacetime we obtain

$$T_{\text{FFAR}} = \frac{1}{8\pi m} \sqrt{1 + x + x^2 + x^3}.
 \tag{3.5}$$

We see that asymptotically far from the black hole T_{FFAR} reduces to the Hawking temperature $T_H = 1/8\pi m$ and then gradually rises to $T_{\text{FFAR}} = 2T_H$ as the horizon is approached. This is in line with physical expectations. In contrast with fiducial observers, who detect high-temperature radiation in the region near the event horizon due to the strong acceleration that is needed to keep them in place, observers in free fall will only detect a smooth rise by an order-one factor in the temperature going from the asymptotic region towards the horizon. Figure 1 plots the two temperatures T_{FID} and T_{FFAR} as a function of the dimensionless radial variable x .

We note that our method gives a physically reasonable answer for T_{FFAR} at all values of $r \geq 2m$. However, the precise numerical value $T_{\text{FFAR}} = 2T_H$ at the event horizon has limited operational meaning. First of all, as was discussed in the introduction, the local free-fall temperature is not a precise notion. On top of this, a freely-falling observer passing through the horizon has only a proper time of order m left before running into the curvature singularity at $r = 0$ and since the characteristic wavelength of thermal radiation at $T \sim 2T_H$ is also of order m the observer cannot measure temperature to better than $\mathcal{O}(1/m)$ precision near the horizon. Although our result for the free-fall temperature is thus only qualitative in the region near the event horizon, it does confirm the expectation expressed in early work of Unruh [13] that an infalling observer will not run into highly energetic particles at the horizon.

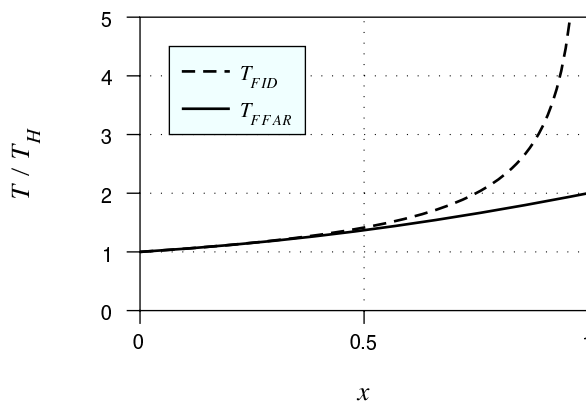


Figure 1: The local temperatures T_{FFAR} and T_{FID} plotted as a function of the dimensionless radial variable $x = 2m/r$. The fiducial temperature T_{FID} blows up at the horizon while the free-fall temperature T_{FFAR} remains finite.

4. AdS-Schwarzschild black holes

Now consider a black hole in 3+1 dimensional spacetime with negative cosmological constant $\Lambda = -3/\ell^2$. The line element is given by (1.2) with

$$f(r) = 1 - \frac{2m}{r} + \frac{r^2}{\ell^2}, \tag{4.1}$$

where m is the mass of the black hole. The event horizon is located at $r = r_H$, where r_H is the real root of $1 - 2m/r + r^2/\ell^2$ and the surface gravity is

$$\kappa = \frac{\ell^2 + 3r_H^2}{2r_H\ell^2}. \tag{4.2}$$

The surface gravity has a minimum value, $\kappa \geq \sqrt{3}/\ell$, and each value above the minimum one is realized for two values of r_H , corresponding to a large AdS black hole with $r_H > \ell/\sqrt{3}$ and a small AdS black hole with $r_H < \ell/\sqrt{3}$. Large AdS black holes with $r_H \gg \ell$ correspond to high-temperature thermal states in a dual gauge theory [14] while small black holes with $r_H \ll \ell$ can be viewed as more-or-less ordinary Schwarzschild black holes in a 'cosmological' background with a negative cosmological constant.

The Hawking temperature (1.1) of AdS black holes on the 'large' branch grows linearly with r_H , and becomes arbitrarily high for very large black holes. As we will see below, this does not mean that the physical temperature measured by an observer in free fall becomes large outside large AdS black holes.

The AdS-Schwarzschild geometry can be globally embedded into a 7-dimensional flat spacetime with the metric $\eta_{IJ} = \text{diag}(-1, 1, \dots, 1, -1)$, which has two time-like dimen-

sions [5]. The embedding functions are given by

$$\begin{aligned}
 Z^0 &= \kappa^{-1} \sqrt{f(r)} \sinh(\kappa t), \\
 Z^1 &= \kappa^{-1} \sqrt{f(r)} \cosh(\kappa t), \\
 Z^2 &= \int dr \frac{\ell^3 + \ell r_H^2}{\ell^2 + 3r_H^2} \sqrt{\frac{r^2 r_H + r r_H^2 + r_H^3}{r^3(r^2 + r r_H + r_H^2 + \ell^2)}}, \\
 Z^6 &= \int dr \frac{1}{\ell^2 + 3r_H^2} \sqrt{\frac{(\ell^4 + 10\ell^2 r_H^2 + 9r_H^4)(r^2 + r r_H + r_H^2)}{r^2 + r r_H + r_H^2 + \ell^2}},
 \end{aligned}
 \tag{4.3}$$

and Z^3, Z^4, Z^5 are three-dimensional spherical coordinates as in equation (3.2).

Calculating the FFAR acceleration as before, one finds the following expression for the free-fall temperature squared,

$$T_{\text{FFAR}}^2 = \frac{1}{16\pi^2 \ell^2} \frac{-4(1+x) + (c^2+1)(c^2+5)x^2 + (c^2+1)^2(1+x+x^2)x^3}{1+x+(c^2+1)x^2},
 \tag{4.4}$$

where $x = r_H/r$ and $c \equiv \ell/r_H$. Figure 2 plots the free-fall temperature squared for black holes on the two branches: A a large one with $c = 0.5$ and a small one with $c = 100$. In both cases T_{FFAR}^2 rises monotonically with x , from a negative value at $x = 0$ to a positive value at the event horizon at $x = 1$. The negative value of T_{FFAR}^2 at spatial infinity reflects the fact that there is no thermal radiation in the empty AdS region asymptotically far away from the black hole. Near the horizon, on the other hand, the free-fall temperature is real valued.

The expression (4.4) for the free-fall temperature simplifies in the two limits $r \rightarrow \infty$ and $r \rightarrow r_H$, i.e. $x \rightarrow 0$ and $x \rightarrow 1$ respectively. For $r \rightarrow \infty$ one obtains

$$T_{\text{FFAR}}^2 \rightarrow -\frac{1}{4\pi^2 \ell^2},
 \tag{4.5}$$

which is precisely the answer found for a geodesic observer in empty AdS space [4]. At the event horizon, $r \rightarrow r_H$, we instead find

$$T_{\text{FFAR}}^2 \rightarrow \frac{1}{4\pi^2 r_H^2}.
 \tag{4.6}$$

For a small AdS black hole with $\ell \gg r_H$ the horizon area reduces to that of a Schwarzschild black hole in asymptotically flat spacetime, $r_H \approx 2m$, and (4.4) reduces to $T_{\text{FFAR}} \rightarrow 2T_H$ at the event horizon. As we move away from the black hole, but remain within the region $r_H < r < \ell$, the free-fall temperature approaches the Hawking temperature. When we get to cosmological length scales $r > \ell$ the geometry approaches that of empty AdS spacetime and T_{FFAR}^2 turns negative. This behavior is evident in the lower plot in figure 2.

On the other hand, for a large AdS black hole with $r_H \gg \ell$ we find that at the event horizon

$$T_{\text{FFAR}} \rightarrow \frac{1}{2\pi r_H} \ll T_H.
 \tag{4.7}$$

The characteristic wavelength of the thermal radiation is then $\lambda \sim 1/T_{\text{FFAR}} \gg \ell$ and at such low temperatures one expects the radiation to be mainly in low angular modes. The

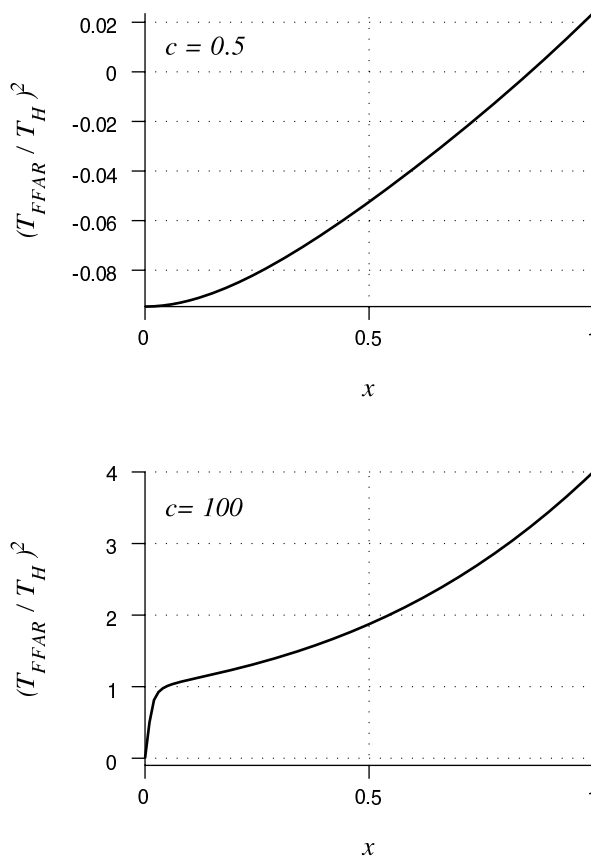


Figure 2: T_{FFAR}^2 plotted in units of T_H^2 for $c = 1/2$ and $c = 100$. A negative value of T_{FFAR}^2 is interpreted as having no thermal radiation in that region.

value of r where the free-fall temperature squared changes sign indicates the size of the radial 'box' that the thermal radiation is confined to. It can be easily verified that T_{FFAR}^2 in equation (4.4) equals zero at the following value of the dimensionless variable x ,

$$x = x_0 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2}{c^2 + 1}}. \tag{4.8}$$

For very large AdS black holes, we can expand in the small parameter $c = \ell/r_H \ll 1$. The r -coordinate location of the zero is $r = r_0$ where

$$\frac{r_0}{r_H} = \frac{1}{x_0} = 1 + \frac{2}{3} \left(\frac{\ell}{r_H}\right)^2 + \mathcal{O}\left(\frac{\ell}{r_H}\right)^4, \tag{4.9}$$

and the spatial proper distance D_0 from the horizon to the point $r = r_0$ is

$$D_0 = \frac{2\sqrt{2}}{3} \left(\frac{\ell}{r_H}\right) \ell + \dots \tag{4.10}$$

The thermal radiation is confined to a layer of thickness $D_0 \ll \ell$ surrounding the horizon of the large AdS black hole. Since the thickness of the layer is much smaller than the characteristic wavelength of thermal radiation at the local free-fall temperature the radiation field

outside the black hole will be in the form of an evanescent wave rather than propagating thermal modes. It therefore appears that an observer in free fall outside a large AdS black hole will not see thermal radiation at all, even if the Hawking temperature of the black hole is very high.

5. Reissner-Nordström black holes

Finally, we calculate the free-fall temperature for a Reissner-Nordström black hole. The line element is given by (1.2) with

$$f(r) = 1 - \frac{2m}{r} + \frac{e^2}{r^2}, \tag{5.1}$$

where m is the mass and e is the electric charge of the black hole. The geometry has two horizons at $r_{\pm} = m \pm \sqrt{m^2 - e^2}$, the zeros of $f(r)$.

It is not necessary to find a global embedding of the Reissner-Nordström spacetime into a higher-dimensional flat spacetime in order to obtain the temperature outside the outer horizon using the embedding method. It is sufficient to find an embedding that covers the region outside the inner horizon [5],

$$\begin{aligned} Z^0 &= \kappa^{-1} \sqrt{f(r)} \sinh(\kappa t), \\ Z^1 &= \kappa^{-1} \sqrt{f(r)} \cosh(\kappa t), \\ Z^2 &= \int dr \sqrt{\frac{r^2(r_+ + r_-) + r_+^2(r_+ + r)}{r^2(r - r_-)}}, \\ Z^6 &= \int dr \sqrt{\frac{4r_+^5 r_-}{r^4(r_+ - r_-)^2}}, \end{aligned} \tag{5.2}$$

where the surface gravity is $\kappa = (r_+ - r_-)/2r_+^2$ and, once again, Z^3, Z^4, Z^5 are three-dimensional spherical coordinates as in equation (3.2). The Z^6 direction is timelike as in the AdS-Schwarzschild case. We note that this embedding is not valid for an extremal black hole but we can deduce the temperature in the extremal case by taking the limit $r_+ \rightarrow r_-$ in the final answer.

The result for the square of the free-fall temperature is

$$T_{\text{FFAR}}^2 = \frac{1}{16\pi^2 r_+^2} \frac{(1 - b)^2(1 + x + x^2 + x^3) - 4bx^4 + 4b^2x^5}{1 - bx}, \tag{5.3}$$

where $x = r_+/r$ and $b = r_-/r_+$. Asymptotically far from the black hole the free-fall temperature reduces to the Hawking temperature,

$$\lim_{x \rightarrow 0} T_{\text{FFAR}} = \frac{r_+ - r_-}{4\pi r_+^2} = T_H. \tag{5.4}$$

In figure 3 the square of T_{FFAR} is plotted for three different values of b .

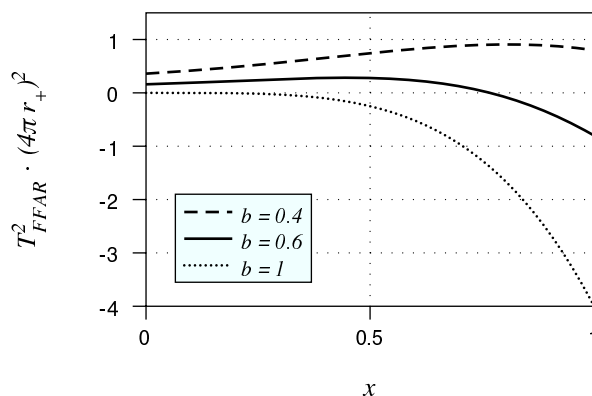


Figure 3: T_{FFAR}^2 for a Reissner-Nordström black hole plotted for three different values of $b = r_-/r_+$. Geometries with $b \leq 0.5$ have positive temperature everywhere outside the black hole but for $b > 0.5$ there is no thermal radiation in a region surrounding the horizon, where T_{FFAR} is purely imaginary.

In the limit $b \rightarrow 0$ the above result for the free-fall temperature reduces to that for the Schwarzschild case, while in the extremal limit $r_{\pm} \rightarrow m$ we find

$$T_{\text{FFAR}}^2|_{\text{extremal}} = -\frac{m^2}{4\pi^2 r^4}. \tag{5.5}$$

As before, a purely imaginary temperature is interpreted as no thermal radiation and this is exactly what is expected for an extremal black hole. Furthermore, $T_{\text{FFAR}}^2|_{\text{extremal}} \rightarrow 0$ as $r \rightarrow \infty$ in agreement with the extremal value of the Hawking temperature, $T_H = 0$.

The curve in the x - b plane where T_{FFAR}^2 changes sign is given by

$$x_0 = \frac{1 - b + \sqrt{1 + 6b - 7b^2}}{4b}, \quad 0 < x_0 \leq 1. \tag{5.6}$$

By analyzing this relation one finds that for $0 \leq b \leq \frac{1}{2}$ there is everywhere positive temperature outside the black hole but when $\frac{1}{2} < b < 1$ the free-fall temperature has a single zero at $r = r_0 > r_+$. Outside $r = r_0$ there is positive temperature but for $r_+ < r < r_0$ the free-fall temperature is purely imaginary, suggesting that there is no thermal radiation in the region close to the horizon of a black hole with high charge-to-mass ratio. This is perhaps not surprising since in the extremal limit the near-horizon geometry approaches $\text{AdS}_2 \times \text{S}^2$ and a freely falling observer in AdS spacetime does not see thermal radiation.

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